

$X \rightarrow \text{diHiggs} \rightarrow 4b$

Boosted Analysis  
Preparations for Run 2

REU work at CERN winter 2015 with the Atlas Exotics group

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20 March 2015



# Outline

- I.) Introduction & short recap on motivation for boosted analysis
- II.) XhhBoosted package development update
- III.) Preliminary results of 13 TeV sample jet cuts study
- IV.) Happening Now & Near future goals : background studies, QCD, ttbar with signal 20 release samples
- V.) Backup slides on some cool things I learned



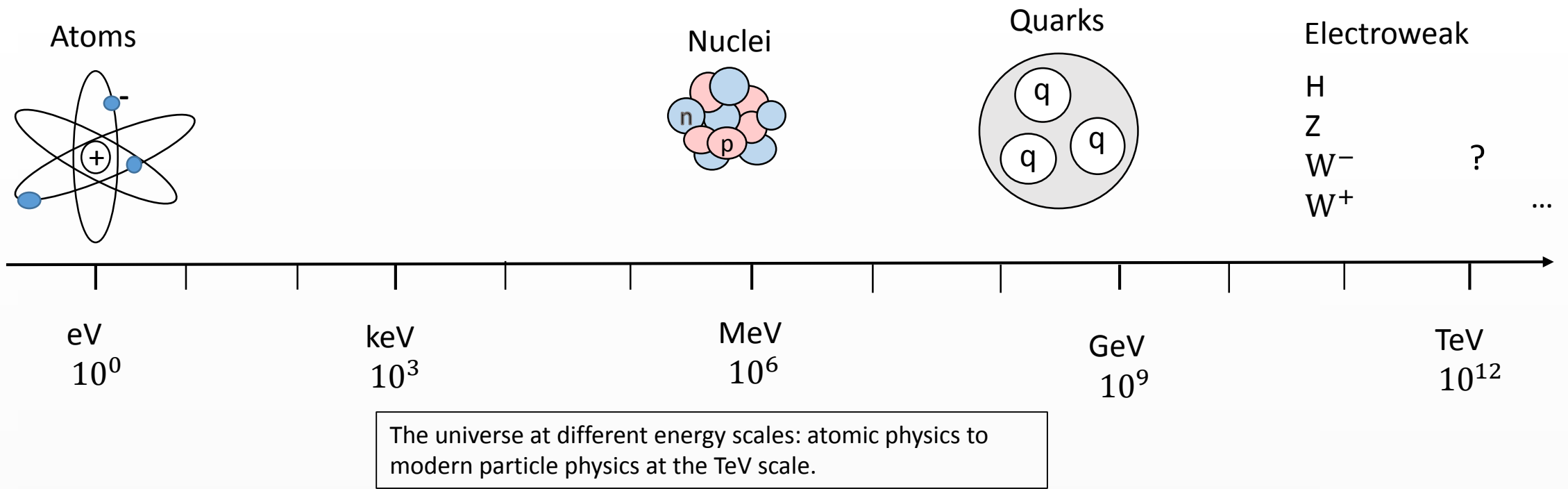
26 March 2015

C. Nelson

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I.)



Discovery of the Higgs boson opens new possibilities of probing the Electroweak scale and the production of heavy particles at the LHC

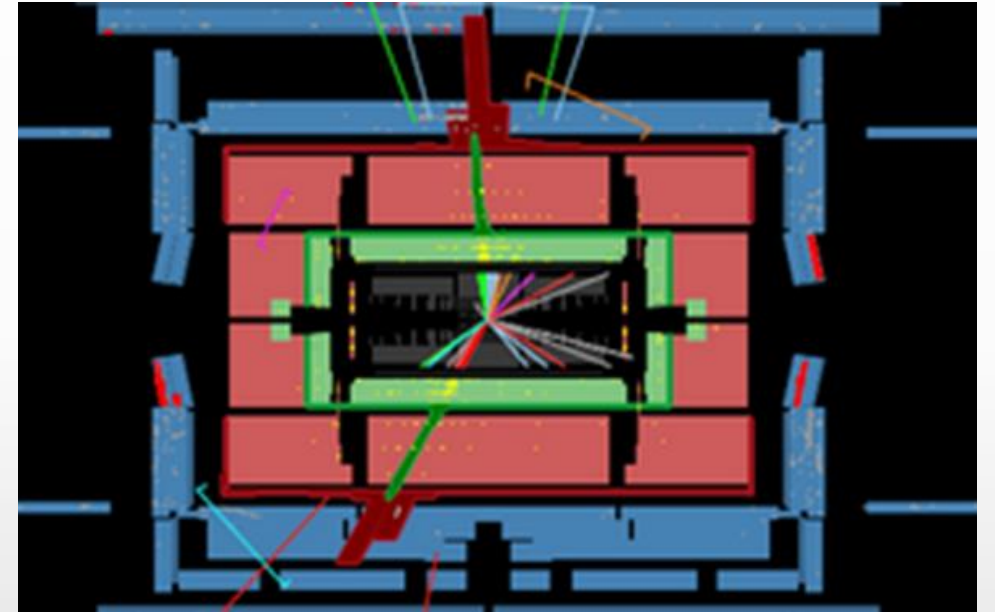
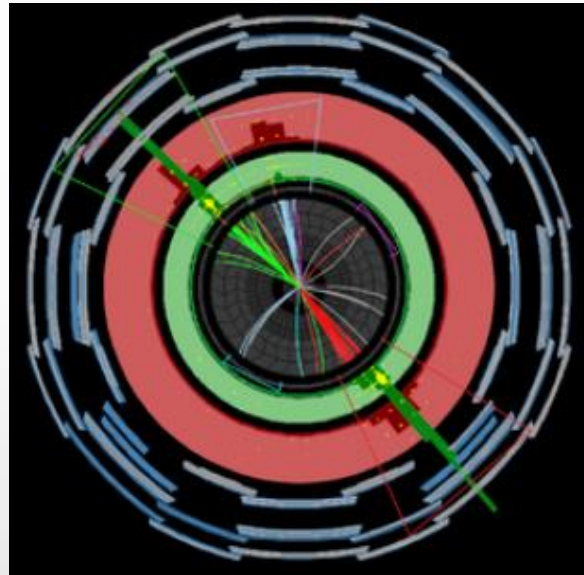
- Use the Higgs boson as a tool for further discovery
- Many new physics models predict significant rates of Higgs pair production
  - New resonances: Kaluza-Klein graviton  $G_{KK} \rightarrow hh$
  - Extended Higgs sector, 2 Higgs doublet model,  $H \rightarrow hh \dots$
  - Non-resonant production: New coloured scalars, direct  $t\bar{t}hh$  vertices...



## II.) Jets in ATLAS

- Transverse view (left): since the colliding partons have no momentum transverse to the beam axis, the jets are produced back to back in the transverse plane and have equal and opposite momenta.
- RZ-view (right): jets are not back to back due to the boost of the final-state system from the net momenta of the colliding partons along the beam axis,  $(x_1 - x_2) \frac{\sqrt{s}}{2}$ .

- “Jets are not just smeared partons”: they are matched by the number of degrees of freedom in the hard radiation that interferes at the amplitude level with the matrix element part of the calculation. (check out perturbation theory in backup)
- “Jets have no existence independent of the algorithm”: jet algorithms do not find jets, they define them.

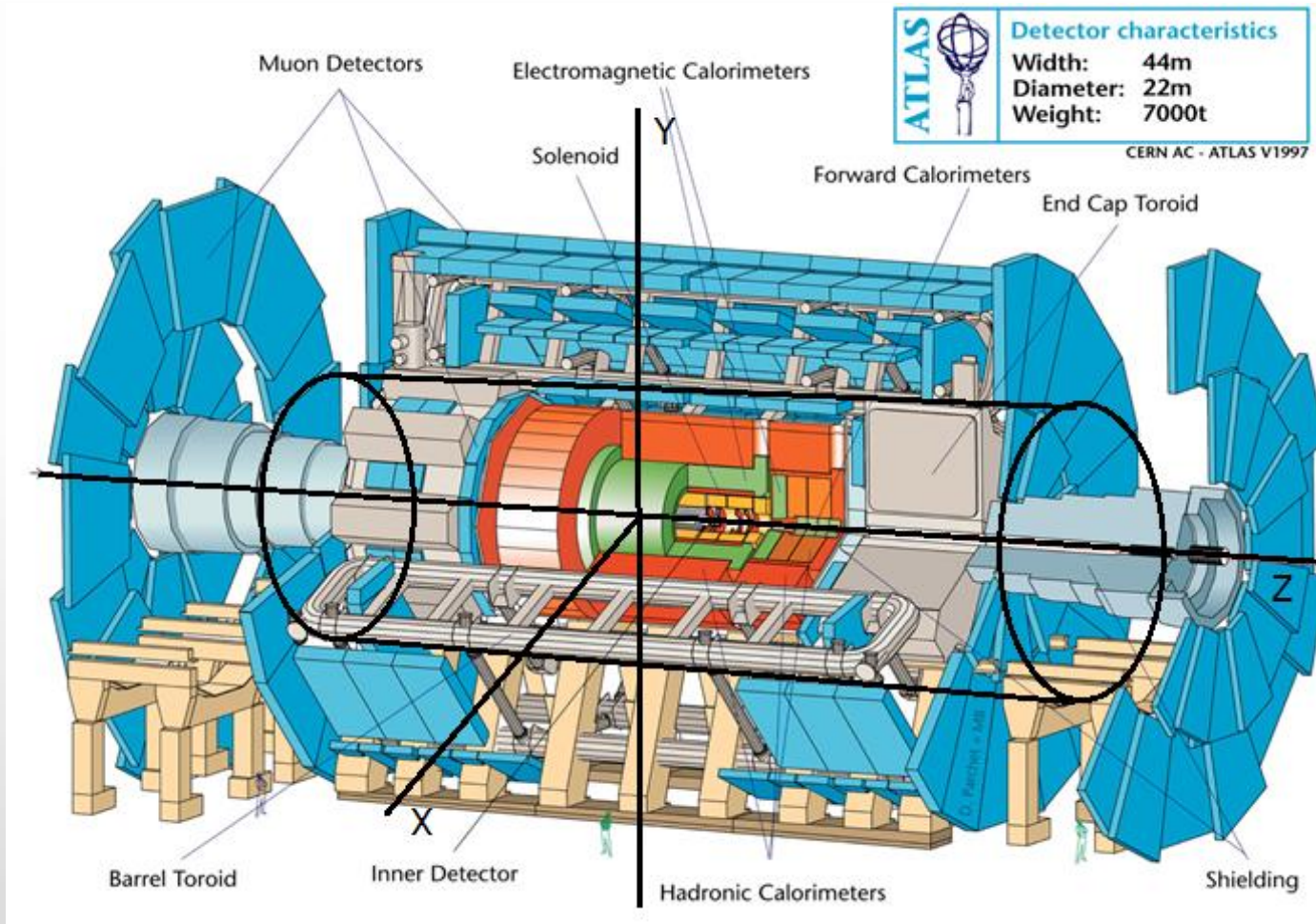


Atlas note: Performance of Jet Algorithms in the  
Atlas Detector High mass jet event with dijet mass 3350 GeV and leading jets with (pT, eta, phi, color) of  
(1800 GeV, -0.51, -0.85, red) and subleading jets with (1470 GeV, -0.05, 2.4, green).  
[https://twiki.cern.ch/twiki/bin/view/AtlasPublic/EventDisplayCONFnotes#Search\\_for\\_New\\_Phenomena\\_in\\_the](https://twiki.cern.ch/twiki/bin/view/AtlasPublic/EventDisplayCONFnotes#Search_for_New_Phenomena_in_the)



# Measured Quantities, and the ATLAS detector... know it... love it

- rapidity ( $y$ ), transverse momentum ( $p_T$ )



- $\theta$  = polar angle measured with respect to the beam line.
- $\phi$  = azimuthal angle measured with respect to x axis.
- Rapidity:  $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$
- Pseudorapidity:  $\eta = -\ln \frac{\theta}{2}$
- Transverse momentum:  $p_T = p \times \sin\theta$
- Transverse energy:  $E_T = E \times \sin\theta$

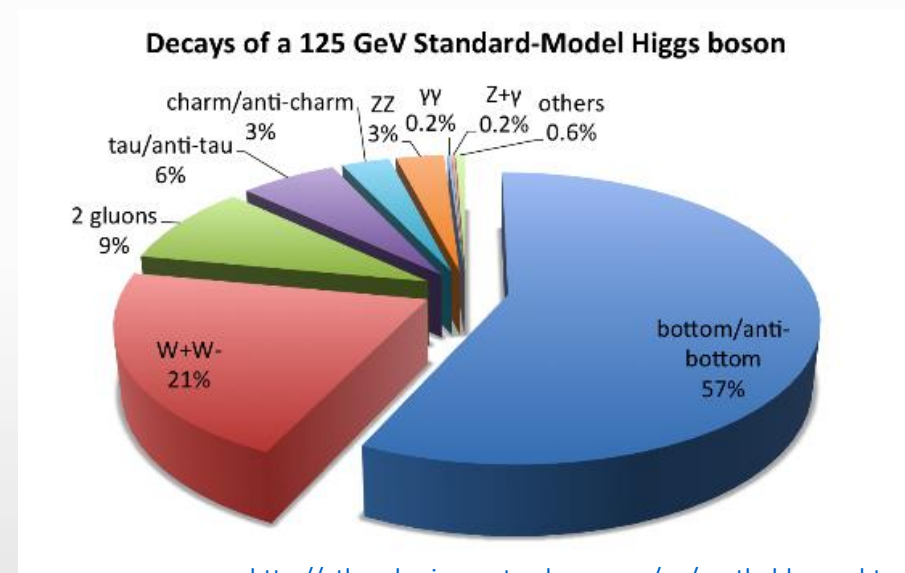
Detector  $\rightarrow$  measurements  $\rightarrow$  particle interactions & cross sections  $\rightarrow$  compare to perturbative QCD fit  $\rightarrow$  compare to PDFs

# Given all this how do we look for new physics and continue to investigate fundamental particles?

- If a b-quark is produced, the hadronization process will create a jet of hadrons, one of which will contain the b-quark, known as b hadrons.
- The b-quarks are relatively long lived with lifetimes of order  $1.5 \times 10^{-12}$ s.

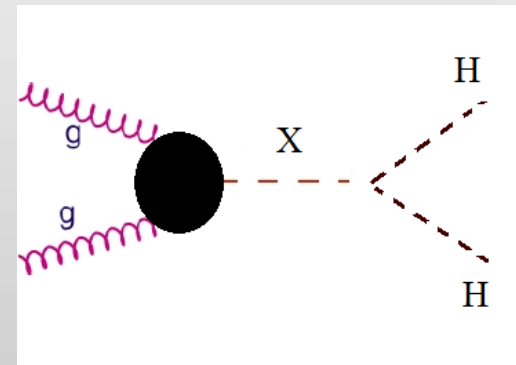
## Discovery of the Higgs boson and measured properties are consistent with the Standard Model

- Branching ration (BR ) :  $BR(j) = \frac{\Gamma_j}{\Gamma}$
- BR of  $H \rightarrow bb \approx 57.7 \%$  at Higgs mass of 125 GeV
- Compare to BR of  $H \rightarrow WW \approx 21.6 \%$
- <https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR>
- This gives an intuition that b flavor tagging and jet reconstruction will help increase sensitivity for searches and analysis



## Leading to opportunities for searches with new physics models

- Many models predict significant Higgs pair production rates at high invariant mass
- Kaluza-Klein graviton  $G_{KK}^*$  in Randall-Sundrum framework
- Warped Extra Dimensions
- 2Higgs Double Model (2HDM)





# Motivation for Boosted Analysis

- Angular separation for 2-body decay products of a heavy particle is approximately  $\Delta R \approx \frac{2m}{p_T}$ ,

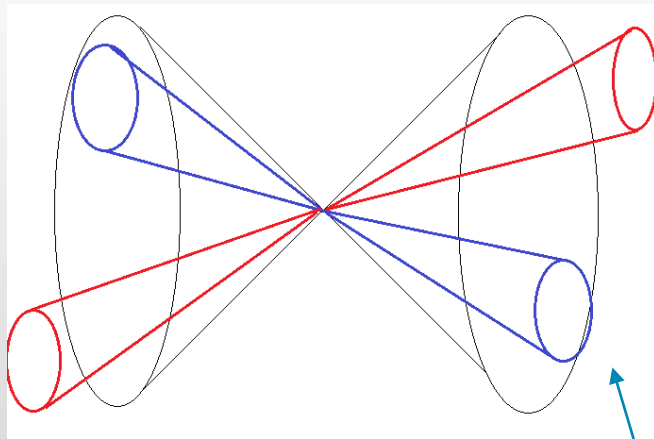
where  $\Delta R = \sqrt{\eta^2 + (\Delta\phi)^2}$ .

- For a Higgs with  $p_T \gtrsim 625 \text{ GeV}$ , the threshold where jets would begin to merge in the detector is  $R = 0.4$

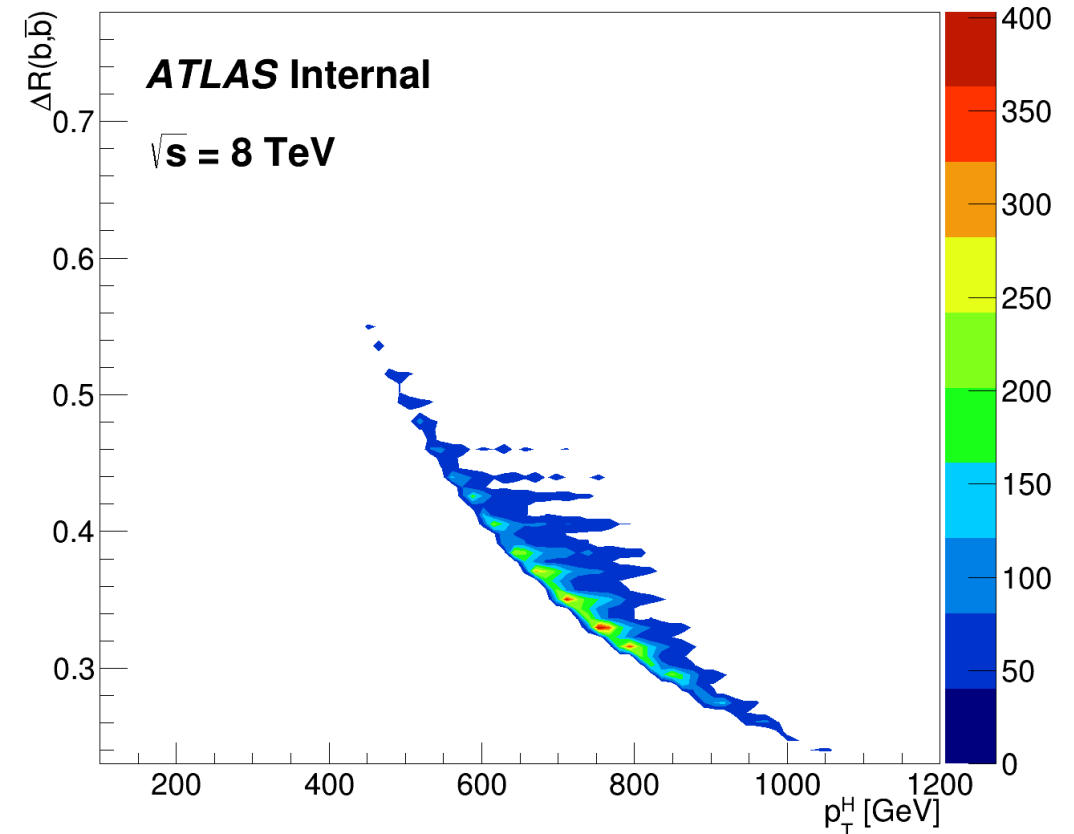
## Analysis Strategy from run1 Boosted Analysis

- Large-R trimmed jets with  $R = 1.0$
- Two b-tagged small R,  $R = 0.3$  track jets ghost associated to each Large-R jet.
- This technique allows Higgs bosons with higher  $p_T$  to be reconstructed.

Leading jet  $p_T > 350 \text{ GeV}$



Subleading jet  $p_T > 250 \text{ GeV}$

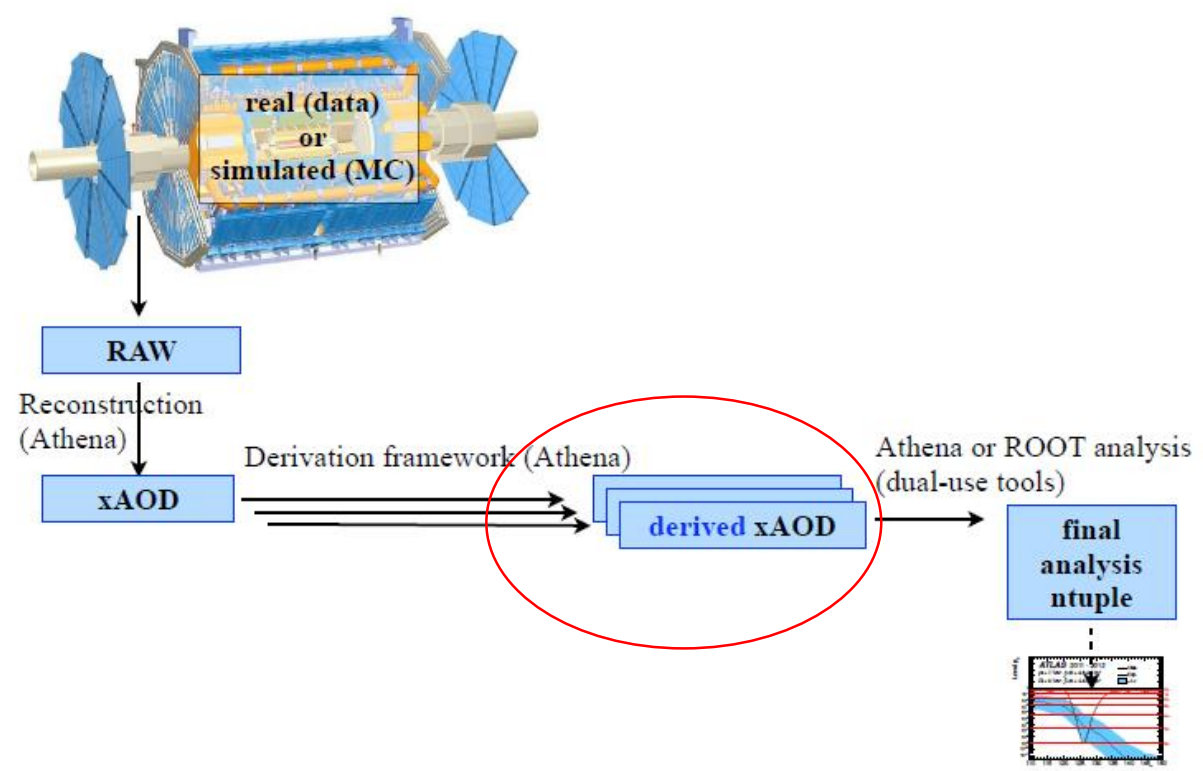


### III.) Analysis :

- Actively developing package with Max : XhhBosted that works with xAODAnaHelpers package

<https://svnweb.cern.ch/trac/atlasphysexo/browser/Physics/Exotic/JDM/hh4b/Run2/Code/XhhBoosted>

The ATLAS Analysis Model



### Schema proposed :

<https://svnweb.cern.ch/trac/atlasphysexo/browser/Physics/Exotic/JDM/hh4b/Run2/Code/XhhBoosted/trunk/util/runXhhBoosted.cxx>

- Run common basic event selection, GRL
- Apply specific Boosted Event Selection functions, seen defined in AnalysisMacros.h here :

<https://svnweb.cern.ch/trac/atlasphysexo/browser/Physics/Exotic/JDM/hh4b/Run2/Code/XhhBoosted/trunk/Root/SetXhhEventCuts.cxx>

- The outputs of this algorithm are:
  - A set of flags in EventInfo to check which cuts were passed
  - A container in Tevent holding xAOD::lparticle links to the 2 large jets, which are linked to 2 small R track jets
  - Proposal to code a common package to make plots/tables out of this information
  - Allow for code flexibility in development and better processing



## The Algorithm :

```
EL::Job basicJobSetup(int argc, char* argv[])
{
    //
    // Grab the configuration file
    //
    std::string configName = "./XhhBoosted/data/steer.config";
    if( argc > 1 ){
        std::cout << "Config file is set to: " << configName << std::endl;
        configName= argv[ 1 ];
    }else{
        std::cout << "Config file defaults to: " << configName << std::endl;
    }
}
```

*Configures input data  
& determines run mode*

```
TEnv* config = new TEnv(configName.c_str());
```

*Configuration files encapsulated  
from steering macros and class  
functions to increase close versatility*

```
//
// Determine the run Mode
//
TString m_runMode = config->GetValue("RunMode", "local");
std::string faxDSName = config->GetValue("FAXDataSetName", "Set DC Name in Config file");
```



```
//  
// Set up the job for xAOD access:  
//  
xAOD::Init().ignore();  
  
//  
// Construct the samples to run on:  
//  
SH::SampleHandler sh;  
  
if(m_runMode.Contains("fax",TString::kIgnoreCase)){  
  std::cout << " Running on fax over. " << faxDSName << std::endl;  
  SH::scanDQ2 (sh, faxDSName);  
}else{  
  std::cout << " Running locally over one file. " << std::endl;  
  // this takes one single file (to be used for testing)  
  
  //  
  // Grab the Path from the config  
  //
```

*Follows the ASG guidelines and is fully supported*

*Here using Sample Handler as recommendation*



```
std::string inputFilePath=config->GetValue("LocalFilePath", "/share/t3data3/johnda/samples/mc14_13TeV.203496.MadGraphPythia8_AU2MSTW2008LO_RSG_hh_bbbb_m500.merge.AOD.e3219_s1982_s2008_r5787_r5853_tid01604035_00/");
```

```
std::string inputFileName = config->GetValue("LocalFileName", "AOD.01604035._000002.pool.root.2");
```

```
SH::DiskListLocal list (inputFilePath);
```

```
SH::scanDir (sh, list, inputFileName);
```

```
}
```

```
//
```

```
// Set the name of the input TTree. It's always "CollectionTree"
```

```
// for xAOD files.
```

```
//
```

```
sh.setMetaString( "nc_tree", "CollectionTree" );
```

```
//
```

```
// Print what we found:
```

```
//
```

```
sh.print();
```





```
//  
// Create an EventLoop job:  
//  
EL::Job ;  
job.sampleHandler( sh );  
  
//  
// Set the number of events  
//  
int nEvents = config->GetValue("maxEvents", -1);  
if(nEvents > 0)  
    job.options()->setDouble(EL::Job::optMaxEvents, nEvents);  
  
return job;  
}
```

*Create Event Loop job  
and  
preliminary job options*

```
void runLocalDriver(EL::Job& job, int argc, char* argv[])  
{
```



```
//  
// Grab the configuration  
//  
std::string configName = "./XhhBoosted/data/steer.config";  
if( argc > 1 ){  
    std::cout << "Config file is set to: " << configName << std::endl;  
    configName= argv[ 1 ];  
}else{  
    std::cout << "Config file defaults to: " << configName << std::endl;  
}
```

```
TEnv* config = new TEnv(configName.c_str());
```

```
//  
// Set the output directory  
//  
std::string submitDir = config->GetValue("SubmitDir", "runXhhBoostedOutput");  
if( argc > 2 ){  
    submitDir = argv[ 2 ];  
}  
std::cout << "Output (Submit) Dir is: " << submitDir << std::endl;
```

*Makes use of Tevent and Tstore to hold and store particles, these are classes from the RootCore skeleton*



```
EL::DirectDriver driver;  
  driver.submit( job, submitDir );  
  return;  
}
```

```
void basicEventSelection(EL::Job& job, std::string name, std::string configFile)  
{  
  BasicEventSelection* baseEventSel = new BasicEventSelection(name, configFile);  
  job.algsAdd( baseEventSel );  
}
```

*The algorithm for the  
Boosted Analysis*

```
void calibrateJets(EL::Job& job, std::string name, std::string configFile)  
{  
  JetCalibrator* jetCalib = new JetCalibrator("jetCalib_"+name,configFile);  
  job.algsAdd( jetCalib );  
}
```

```
void selectCaloJets (EL::Job& job, std::string name, std::string configFile)  
{  
  JetSelector* jet_selection = new JetSelector ("selCaloJets_"+name,configFile);  
  job.algsAdd( jet_selection );  
}
```





```
void setXhhEventCuts(EL::Job& job, std::string name, std::string configFile)
{
    SetXhhEventCuts* setXhhEventCuts = new SetXhhEventCuts(name, configFile);
    job.algsAdd( setXhhEventCuts );
}
```

*Right now most  
testing is being  
done in  
setXhhEventCuts*

```
void eventCount(EL::Job& job, unsigned int eventsPerPrint = 100, bool debug=false)
{
    EventCount* eventCount = new EventCount();
    eventCount->m_eventsPerPrint = eventsPerPrint;
    job.algsAdd( eventCount );
}
```

*// to be implemented still*

```
/*void calibrateMuons(EL::Job& job, std::string name, std::string configFile)
{
    MuonCalibrator* muonCalib = new MuonCalibrator("muonCalib_"+name, configFile);
    job.algsAdd( muonCalib );
}
```

```
void selectMuons(EL::Job& job, std::string name, std::string configFile)
{
    MuonSelector* muon_selection = new MuonSelector ("muons_"+name, configFile);
    job.algsAdd( muon_selection );
}*/
```



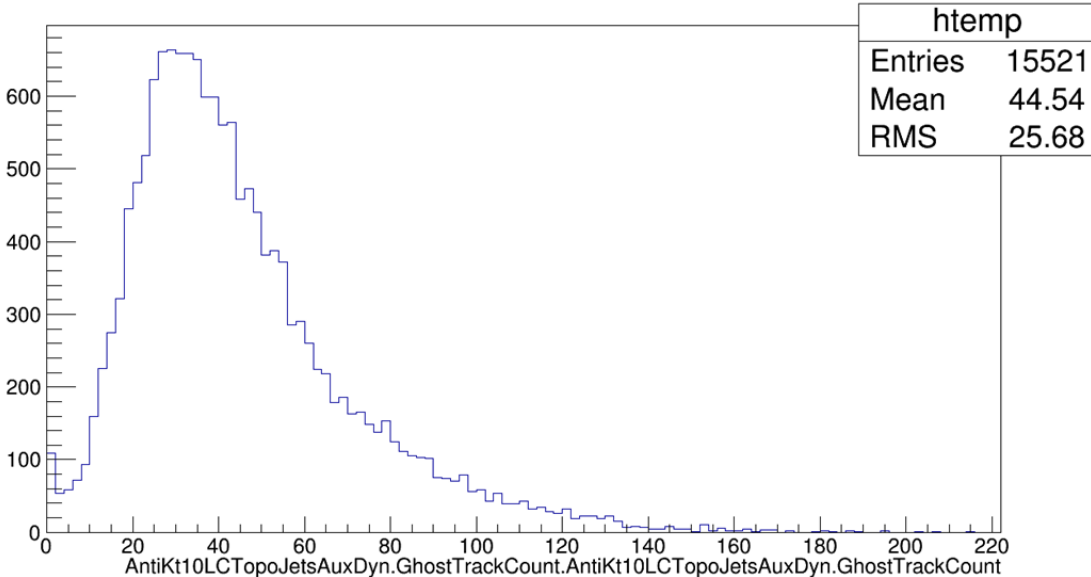
## Signal sample using:

- Mass of the  $G_{KK}$  at resonance of 500 GeV (working on obtaining different mass resonances presently)
- DC14 release 19,  $\sqrt{s} = 13$  TeV
- EventCount and SetXhhEventCuts are the classes accessed by macros

- Console output after running XhhBoosted :

```
Info in <BasicEventSelection::finalize(): Number of events = 5000
Info in <JetSelector::finalize(): selCaloJets_selCaloJets
Info in <JetSelector::histFinalize(): Filling_cutflow
Total Events Processed... 5000 Events (Rate: 220.751 Hz)
Cut PassAssoTrackJets 315 0.063
Cut PassAtLeast2Jets 538 0.1076
Cut PassLeadJetPt 343 0.0686
Cut PassSubLeadJetPt 538 0.1076
Cut PassTrackJetEta 45 0.009
Cut PassTrackJetPt 45 0.009
Cut PassXhhTrig 0 0
Info in <BJetSelector::finalize(): Deleting tool instances...
Info in <BJetSelector::finalize(): Deleting tool instances...
DiJets: Number of passed di-jets: 2576980377/(858993466)
Info in <JetSelector::histFinalize(): Calling histFinalize
```

AntiKt10LCTopoJetsAuxDyn.GhostTrackCount.AntiKt10LCTopoJetsAuxDyn.GhostTrackCount



Directly from signal sample, count of events after AntiKt10LCTopo algorithm applied.

- AntiKt R = 1.0 jet trimming
- Pass 2 Jets : Leading and subleading with  $p_T > 350$  GeV and  $p_T > 250$  GeV respectively, both with  $\eta < 2.0$  (between fat jets)
- Tracks jet with R = 0.3 are ghost associated to large R ungroomed jets
- Track jets must have  $p_T > 20$  GeV and  $\eta < 2.5$  (trimmed calorimeter jet and originate from primary vertex)
- Last step to be implemented before b-tagging delta  $\eta < 1.7$  cut between 2 fat jets



Console output:

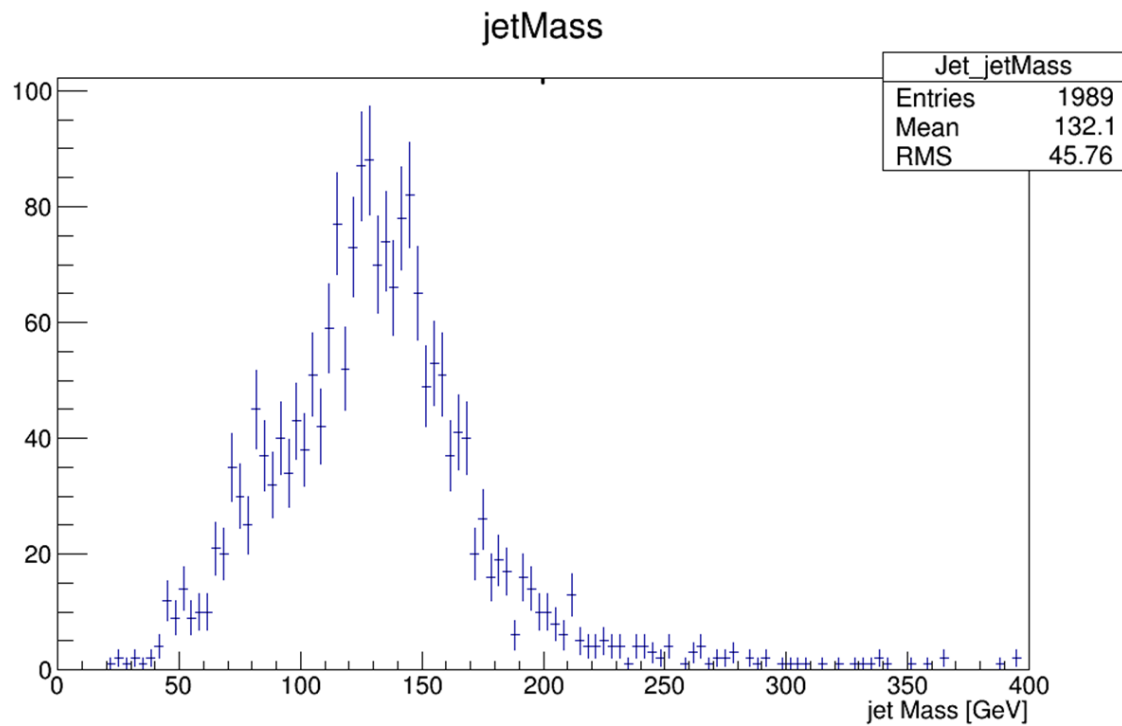
\*printing out some information about the jets that are in the sample and that pass the cuts.

\*actively working on getting different mass samples

```
Info in <SetXhhEventCuts::execute(): jet pt = 386.99 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.26
Info in <SetXhhEventCuts::execute(): jet pt = 333.15 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.54
Info in <SetXhhEventCuts::execute(): trackJetPt1 = 12.92 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt2 = 174.34 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt3 = 12.92 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt4 = 174.34 GeV
Info in <SetXhhEventCuts::execute(): jet pt = 267.74 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.77
Info in <SetXhhEventCuts::execute(): jet pt = 257.73 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.36
Info in <SetXhhEventCuts::execute(): jet pt = 368.73 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.85
Info in <SetXhhEventCuts::execute(): jet pt = 287.15 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.18
Info in <SetXhhEventCuts::execute(): trackJetPt1 = 2.11 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt2 = 3.32 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt3 = 2.11 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt4 = 3.32 GeV
Info in <SetXhhEventCuts::execute(): jet pt = 682.92 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.16
Info in <SetXhhEventCuts::execute(): jet pt = 671.33 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 1.22
Info in <SetXhhEventCuts::execute(): trackJetPt1 = 15.11 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt2 = 14.13 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt3 = 15.11 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt4 = 14.13 GeV
Info in <SetXhhEventCuts::execute(): jet pt = 496.34 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.09
Info in <SetXhhEventCuts::execute(): jet pt = 362.89 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.52
Info in <SetXhhEventCuts::execute(): jet pt = 379.72 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.06
Info in <SetXhhEventCuts::execute(): jet pt = 354.35 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.68
Info in <SetXhhEventCuts::execute(): trackJetPt1 = 10.03 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt2 = 2.56 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt3 = 10.03 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt4 = 2.56 GeV
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Warning in <xAOD::THolder::getAs>: Trying to retrieve ConstDataVe
et_v1> pointer
Info in <SetXhhEventCuts::execute(): jet pt = 382.62 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 1.09
Info in <SetXhhEventCuts::execute(): jet pt = 287.25 GeV
Info in <SetXhhEventCuts::execute(): jet eta = 0.62
Info in <SetXhhEventCuts::execute(): trackJetPt1 = 7.93 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt2 = 7.38 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt3 = 7.93 GeV
Info in <SetXhhEventCuts::execute(): trackJetPt4 = 7.38 GeV
```

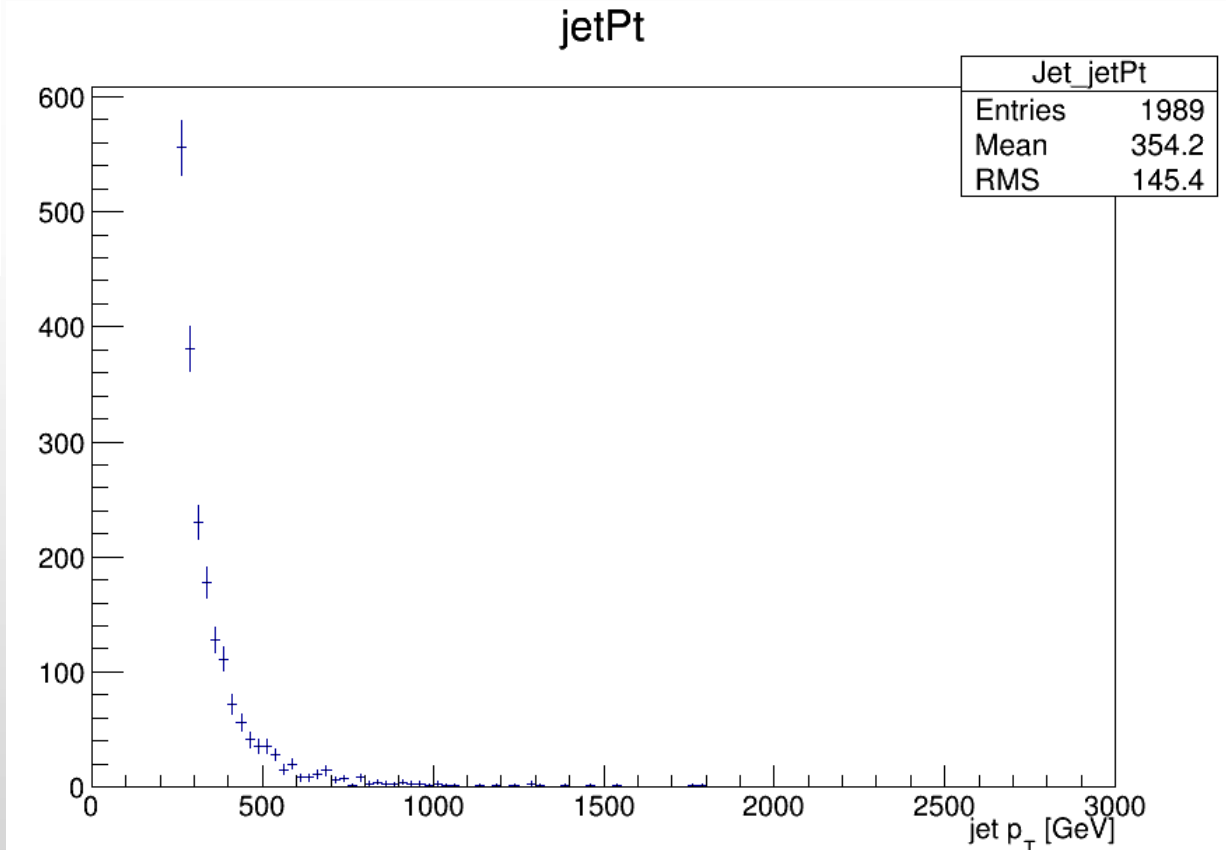
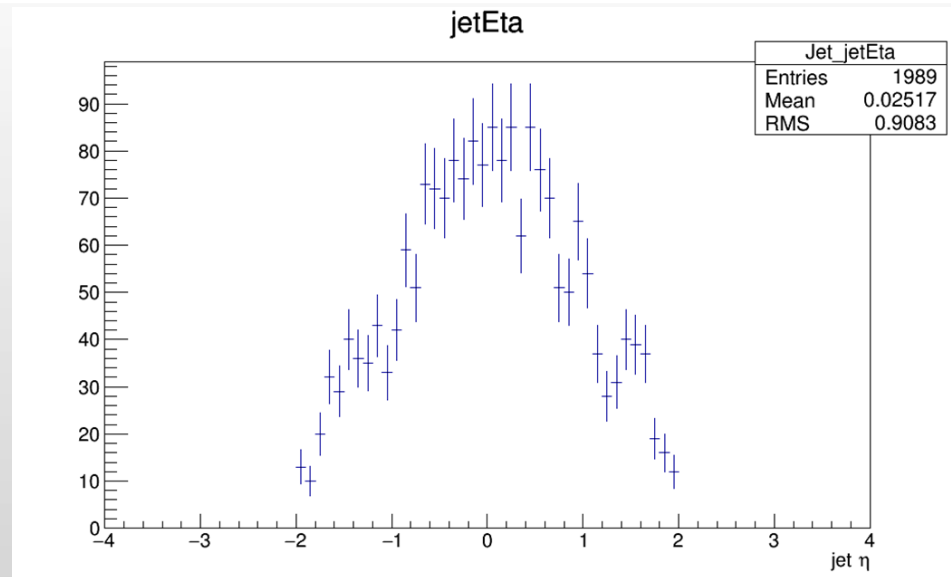






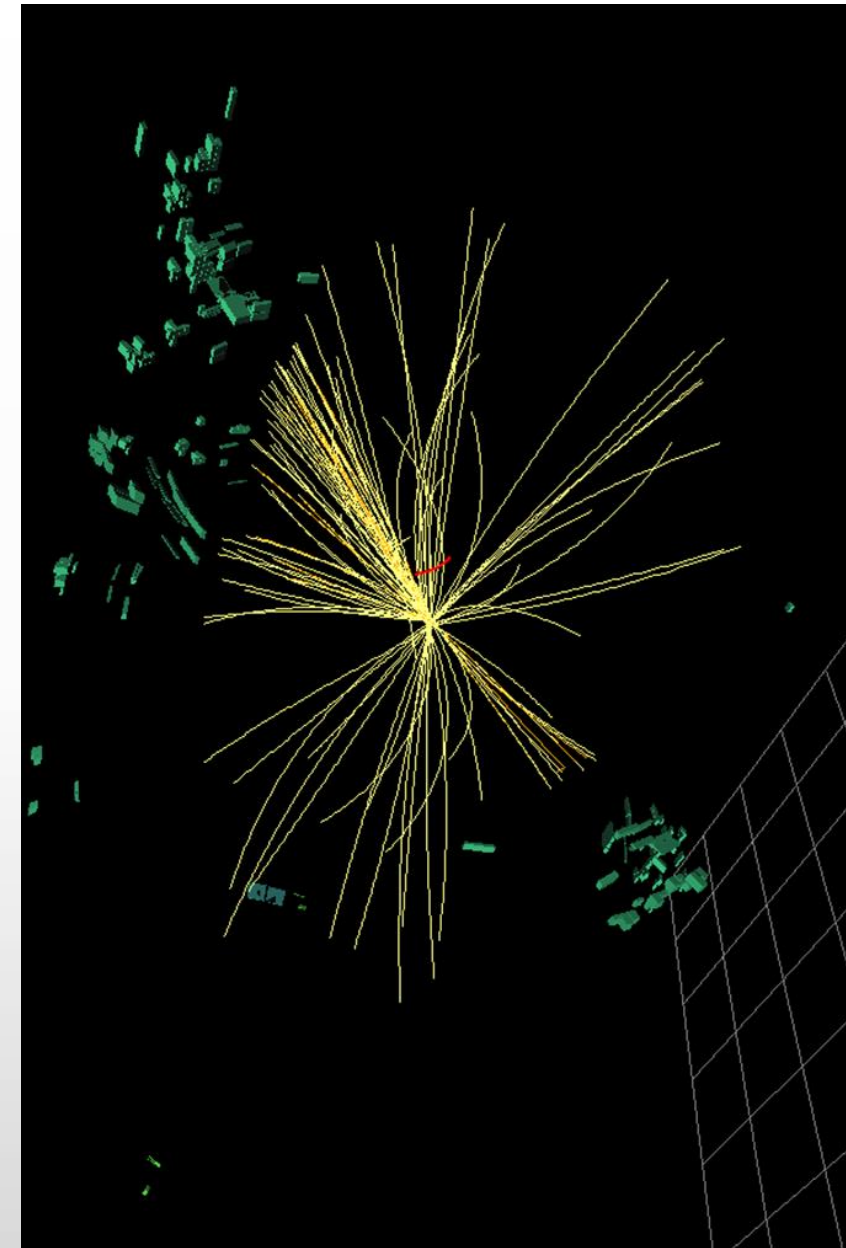
After running package:

- After Leading  $p_T > 350$  GeV cut
- 1989 out of 5000 events pass the cut
- Jet mass peaks about 8 GeV greater than the Higgs mass
- Preliminary efficiency for 500 GeV resonance mass approximately 40 %
- Statistical error and different masses will be studied



## IV.) Happening now & Near Future Goals :

- Working with Max on building XhhBoosted package, this is an xAOD Event loop based algorithm to be complete by May
- Working with Reina Camacho & Michael Kagan in the Boosted X  $\rightarrow$  bb tagging group (collaboration including Max and many others)  
<https://twiki.cern.ch/twiki/bin/viewauth/AtlasProtected/BoostedHiggsToBBTagging>
- Study performance for anti-kt trimmed  $R = 1.0$ ,  $R_{\text{sub}} = 0.3$  and  $R_{\text{sub}} = 0.2$ , and  $f = 5\%$  study performance at 13 TeV : signal versus QCD and top background.
- This is in alignment with what is needed in Diboson  $\rightarrow$  4b Exotics Analysis studies as well. (the goal is not to create more work, but to work more efficiently as a collaboration and to help ensure an excellently prepared and tested analysis for run2)



Just some playing around with the useful features of vp1 for visualizing jets in hadronic collisions : yellow are tracks of particle collisions, red are muon tracks, and green are calorimeter energy deposits. One can get a visual sense of where jets may be reconstructed.

- **Plot efficiency and define signal, control, sideband regions**
- **Rel20 13TeV samples to work with B-tagging**
- **Background QCD and ttbar studies**



## Boosted X--> bb tagging

- ↓ [Basic Information](#)
- ↓ [Introduction](#)
- ↓ [Guidelines and Recommendations](#)
- ↓ [2015 Pre-recommendations Campaign](#)
  - ↓ [Standard Analysis](#)
  - ↓ [Samples](#)

<https://twiki.cern.ch/twiki/bin/viewauth/AtlasProtected/BoostedHiggsToBBTagging>

Task ID	Task description	Assignment	Status	Deadline
<b>Short term tasks</b>				
A	B-tagging - Define working points and study performance for b-tagging using track jets			
A-1	Optimisation and performance at 8 and 13 <a href="#">TeV</a>	M. Kagan, Q. Zeng	Complete: <input type="text"/>	Before Data
A-2	b-tagging calibration and extrapolation from 8 to 13 <a href="#">TeV</a>	Unassigned?	Complete: <input type="text"/>	Before data
B	Jet algorithms - Study performance for anti-kt trimmed R=1.0, R <sub>sub</sub> =0.2 and f=5%			
B-1	Performance at 8 and 13 <a href="#">TeV</a> : signal vs QCD/top background	Christina Nelson	Complete: <input type="text"/>	Before Data
B-2	JMS and JES using QCD jets at 13 <a href="#">TeV</a>	C. Delitzsch	Complete: <input type="text"/>	Before data
B-3	Cross-calibration using QCD jets 13 <a href="#">TeV</a> samples	Unassigned?	Complete: <input type="text"/>	Before data
B-4	JES and JMS flavour uncertainty from H->bb using 13 <a href="#">TeV</a> samples	Unassigned?	Complete: <input type="text"/>	Before data
C	Substructure - Using substructure variables adds on top of b-tagging?			
C-1	Performance at 8 and 13 <a href="#">TeV</a> before and after b-tagging	Q. Zeng, Chiao-Ying Lin, T. Lenz, S. Ballensiefen	Complete: <input type="text"/>	Before Data





*Special thanks to my mentors :  
Massimiliano Bellomo and  
Stephane Willocq*



# ATLAS EXPERIMENT





Petzi tickets.ch  
NELSON CHRISTINA  
N° 3047400234656

PEACE - THE FAMILY RAIN - POLAR CIRCLES - THE BURNING BEGGARS

LES HIVERNALES  
SATURDAY 28/02/2015

START: 21:30  
DOORS: 20:30  
PRICE: CHF 28.00



Petzi tickets

LES HIVERNALES




C. Nelson

Monts Jura

VALLEE DE CHAMONIX MONT-BLANC

1700 km de Pistes / Skiway  
22 000 m de Descentes / tél.纜車  
20 Stations de Ski / 150 km de Pistes  
Descente gratuite pour tous /  
Non interrompue les jours  
des Fêtes

à 2000 km de Paris / Espagne  
à 100 km de la Suisse / 100 km de la France  
à 100 km de la France / 100 km de la France  
à 100 km de la France / 100 km de la France

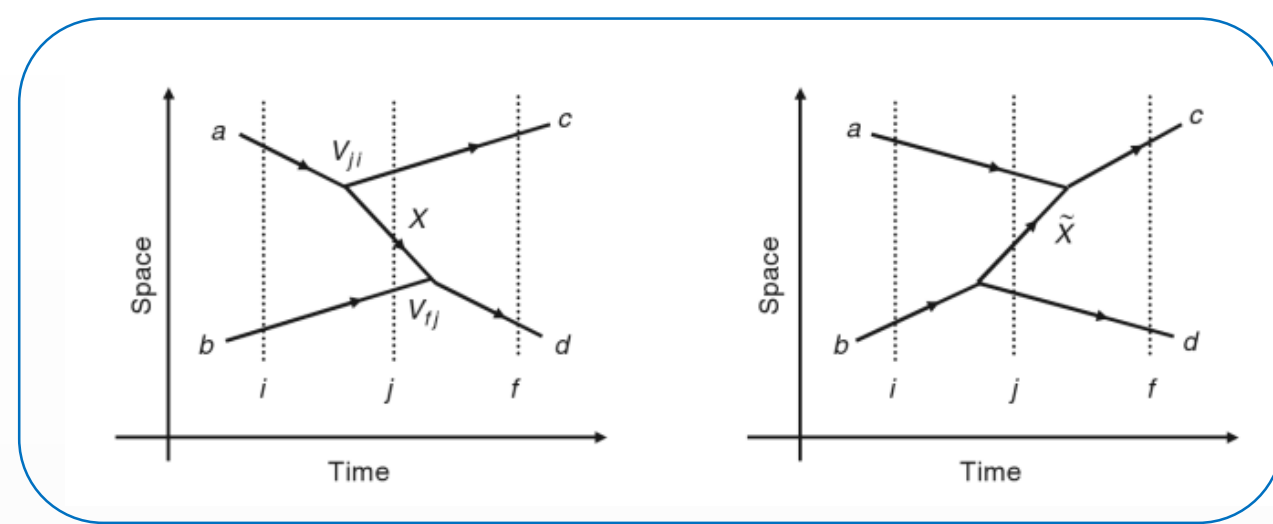



# Cool & useful things I learned



# Perturbation Theory

- The process of understanding interactions between particles by the exchange of force carrying gauge bosons.
- Two possible time-orderings for the process  $a+b \rightarrow c+d$  which can occur via an intermediate state corresponding to the exchange of particle  $X$ .



Initial state  $|i\rangle$  corresponds to particles  $a + b$ , intermediate state  $|j\rangle$  corresponds to  $c + b + X$ , and final state  $|f\rangle$  corresponds to  $c + d$ .

- Left diagram: particle  $a$  can be thought of as emitting the exchanged particle  $X$ , and then at a later time  $X$  is absorbed by  $b$ .
- Right diagram: particle  $b$  emitting the exchanged particle  $\tilde{X}$ , which is then absorbed by  $a$ .  $\tilde{X}$  has the same mass as  $X$  but opposite charge.
- Rewriting the four-momentum of the exchanged particle  $X$ , as  $q = p_a - p_c$  we find that

$$M_{fi} = \frac{g_a g_b}{q^2 - m_X^2}.$$

***This remarkable result shows that the interaction matrix, depending on four momentum, is Lorentz invariant and is the sum of different time ordered processes !***





# Parton Distribution Functions :

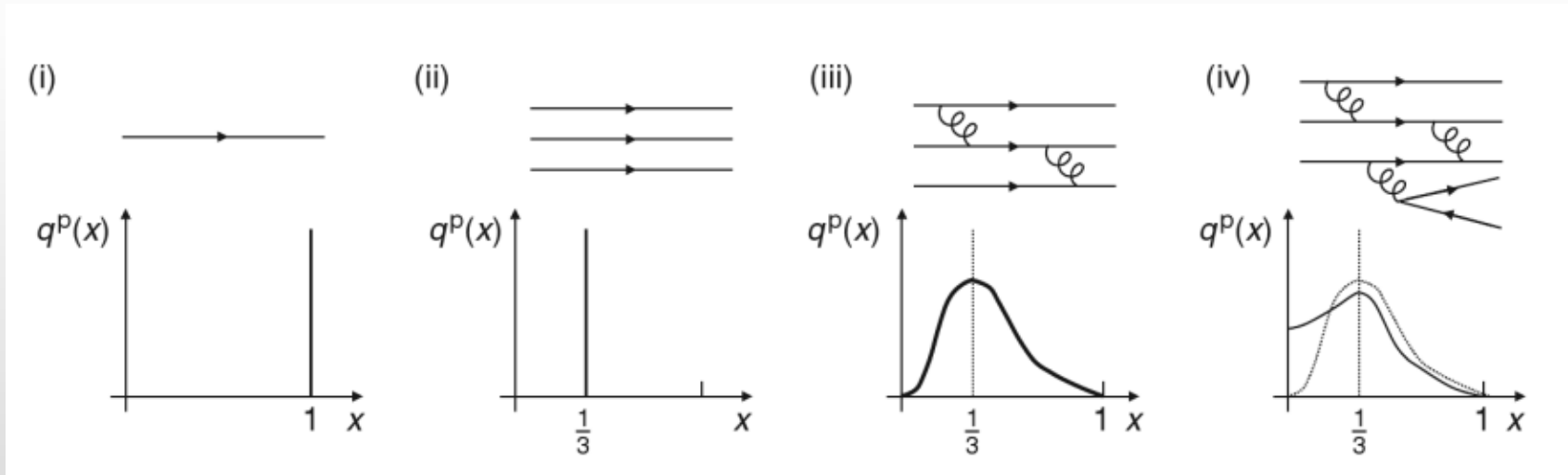
- PDFs are obtained from experiment.

The dynamics of quarks interacting with each other inside a proton through the exchange of gluons results in a distribution of quark momenta within the proton. The distributions are expressed in terms of PDFs. For example, the bottom-quark PDF for the proton  $b^p(x)$  is defined such that

$$b^p(x)\delta x$$

represents the number of bottom quarks within the proton with momentum fraction between  $x$  and  $x + \delta x$ .

Illustrating a few possible forms of PDFs:



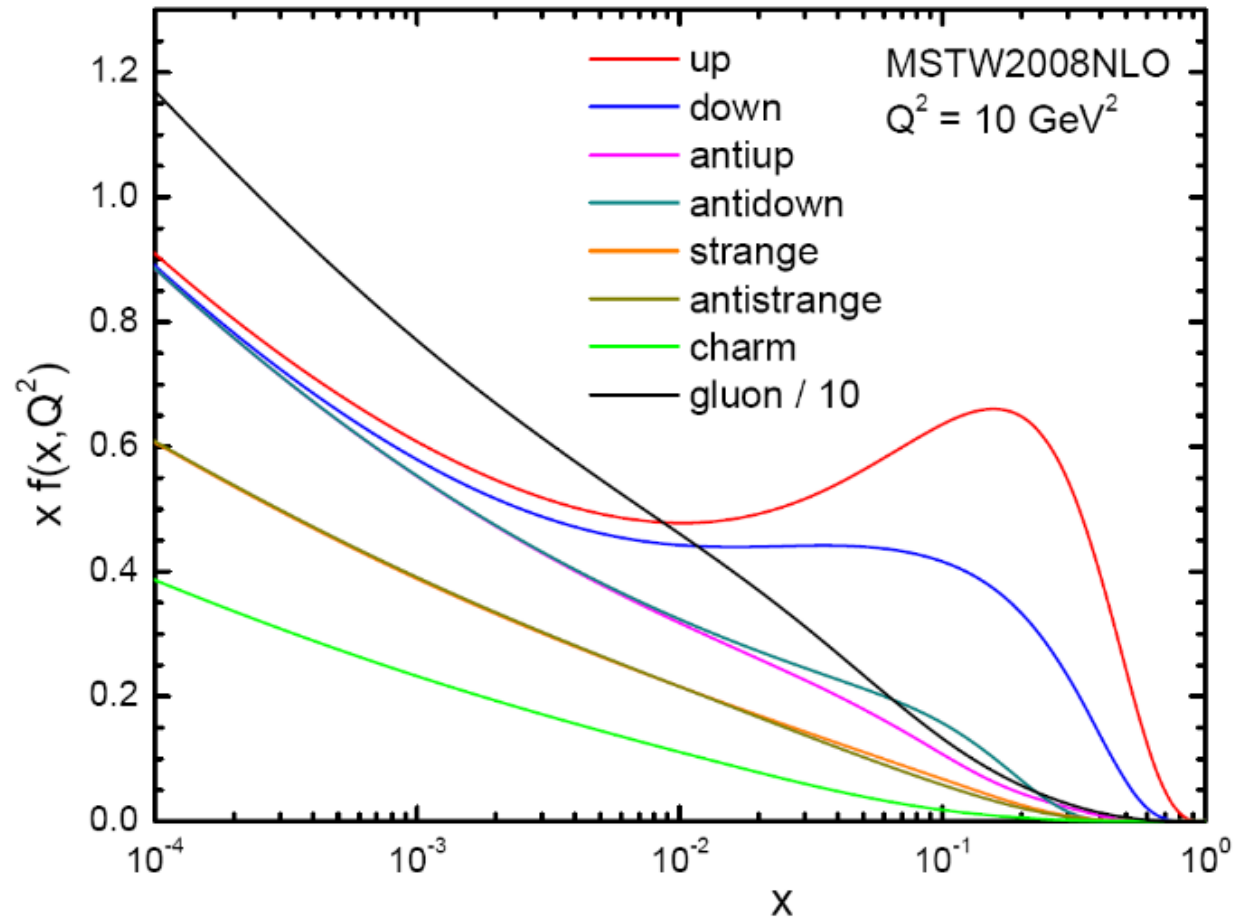
- (i) Proton a single point like particle which carries all the momenta of proton, at Dirac delta function  $x = 1$ .
  - (ii) Proton contains 3 static quarks each carrying  $\frac{1}{3}$  the momenta of the proton, at Dirac delta function  $x = \frac{1}{3}$
  - (iii) Three quarks interact with each other and delta-function at  $x = \frac{1}{3}$  is smeared out as the quarks exchange momentum
  - (iv) Higher-order processes, e.g. virtual quark pairs being produced from gluons inside the proton, tend to result in an enhancement of the PDFs at low  $x$ , reflecting the  $\frac{1}{q^2}$  nature of the gluon propagator.
- Thomson pp.192.

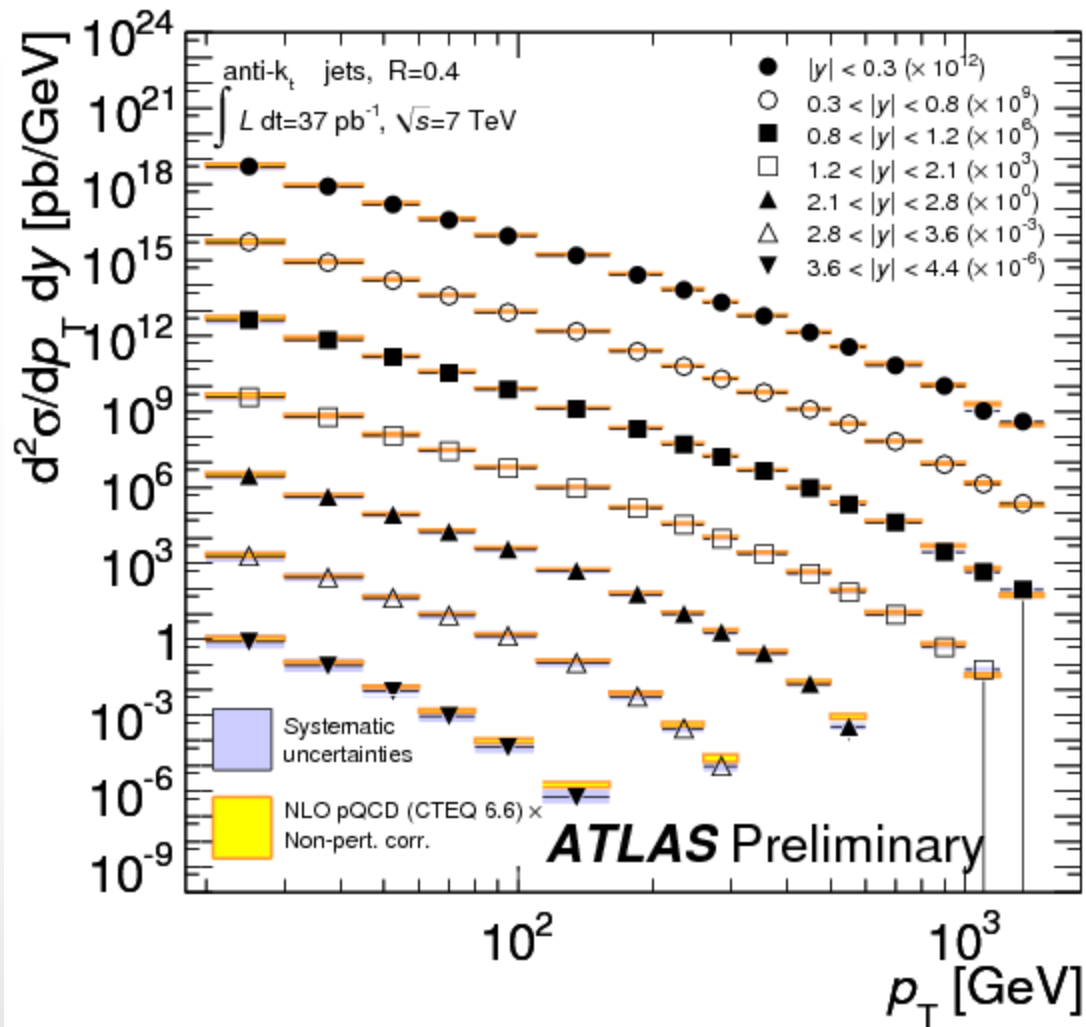
**Note: shape of function as approaching higher order processes**



# Parton Distribution Functions

PDFs reflect the underlying structure of the proton. At present they cannot be calculated from first principles. This is because the theory of QCD has a large coupling constant, and perturbation theory cannot be applied. Therefore the PDFs are extracted from measurements of the structure functions in deep inelastic scattering experiments (fixed target experiments) and other experimental data.

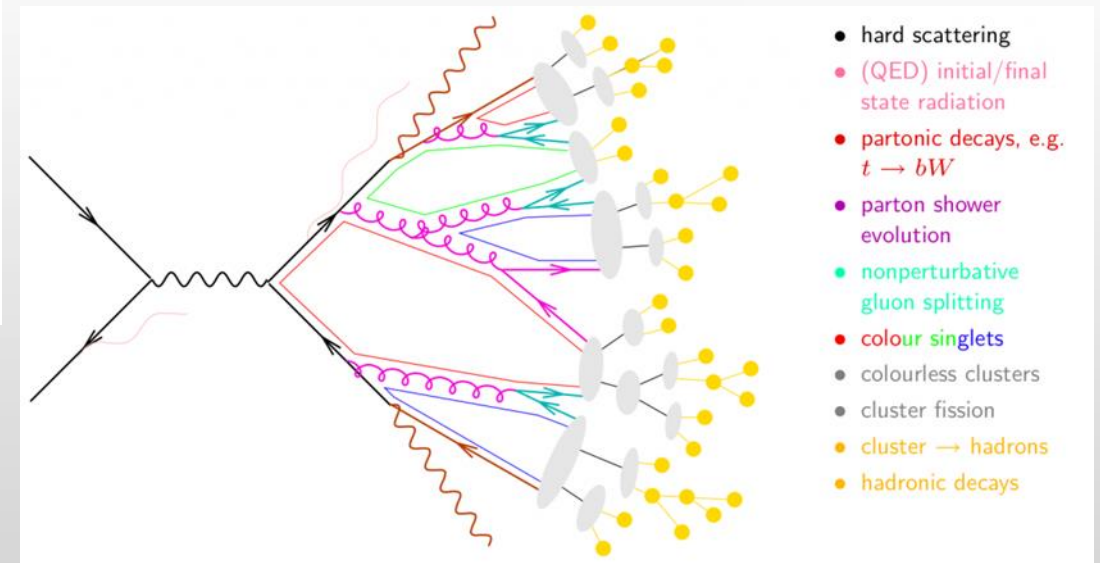




<http://inspirehep.net/record/945445/plots>

- Inclusive jet double-differential cross section as a function of jet  $p_T$  compared to next-leading-order (NLO) pQCD predictions, using current knowledge of PDFs corrections have been applied.

- QCD is found to provide excellent description of jet phenomena in hadron hadron collisions.
- Owing to the nature of QCD, quarks are always confined in hadrons. However, in high energy collisions, it is quarks that are produced, not hadrons.
- As a result of QCD interaction, the strong interaction field between the quarks produce further quarks and antiquarks, known as hadronization
- From hadronization, each quark produced in a collision produces a jet of hadrons.



## Derivation of Time-ordered perturbation theory to the second order (for fun 😊) :

In quantum mechanics, the transition rate between an initial state,  $i$ , and final state,  $f$ , is given by Fermi's golden rule  $\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_f)$ , where  $T_{fi}$  is the transition matrix element, given by the perturbation expansion

$$T_{fi} = \langle f|V|i\rangle + \sum_{j \neq i} \frac{\langle f|V|j\rangle \langle j|V|i\rangle}{E_i - E_j} + \dots$$
 Referring to left diagram on slide 3 & 4,

the quantum mechanical perturbation expansion is

$$T_{fi}^{ab} = \frac{\langle f|V|j\rangle \langle j|V|i\rangle}{E_i - E_j} = \frac{\langle d|V|X + b\rangle \langle c + X|V|a\rangle}{(E_a + E_b) - (E_c + E_X + E_b)}$$
, where  $T_{fi}^{ab}$  is the time ordering where the interaction between  $a$

and  $X$  occurs before that between  $X$  and  $b$ . Allowed by the energy-time uncertainty relation of quantum mechanics  $E_j \neq E_i$ . The non-invariant matrix element  $V_{ji} = \langle c + X|V|a\rangle$  is related to the Lorentz Invariant (LI) matrix element  $M_{ji}$ , by normalizing the phase space wave function. Then,

$$V_{ji} = M_{ji} \prod_k (2E_k)^{-\frac{1}{2}}$$

and the index  $k$  runs over the particles involved. Here, we have

$$V_{ji} = \langle c + X|V|a\rangle = \frac{M_{a \rightarrow c+X}}{(2E_a 2E_c 2E_X)^{\frac{1}{2}}}$$

where  $M_{a \rightarrow c+X}$  is the LI matrix element for the fundamental interaction  $a \rightarrow c + X$ . The requirement of LI on  $M_{a \rightarrow c+X}$  places strong constraints on its possible mathematical structure. To give an example of particle exchange, the simplest possible Lorentz-Invariant coupling is here assumed to be a scalar. In this case, the matrix element is  $M_{a \rightarrow c+X} = g_a$ . Thus,

$$V_{ij} \langle c + X|V|a\rangle = \frac{g_a}{(2E_a 2E_c 2E_X)^{\frac{1}{2}}}$$

C. Nelson





Where the magnitude of the coupling constant  $g_a$  is a measure of the strength of the scalar interaction. Similarly we can express  $g_b$  as the coupling strength in the  $b + X \rightarrow d$  interaction vertex,

$$V_{fj} = \langle d|V|X + b \rangle = \frac{g_b}{(2E_b 2E_d 2E_X)^{\frac{1}{2}}}.$$

Therefore, with the assumed scalar interaction, the second-order term in the perturbation series is

$$T_{fi}^{ab} = \frac{\langle d|V|X + b \rangle \langle c + X|V|a \rangle}{(E_a + E_b) - (E_c + E_X + E_b)} = \frac{1}{2E_X} \cdot \frac{1}{(2E_a 2E_b 2E_c 2E_d)^{\frac{1}{2}}} \cdot \frac{g_a g_b}{(E_a - E_c - E_X)}.$$

The LI matrix element for the process  $a + b \rightarrow c + d$  is related by the corresponding transition matrix element by

$$M_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{\frac{1}{2}} T_{fi}^{ab}$$

and thus,

$$M_{fi}^{ab} = \frac{1}{2E_X} \cdot \frac{g_a g_b}{(E_a - E_c - E_X)}.$$

The matrix element  $M_{fi}^{ab}$  is Lorentz invariant as a scalar interaction and defined in terms of wave functions with an appropriate LI normalization. For this second-order process in perturbation theory momentum is conserved at the interaction vertices but energy is not. Also, the exchanged particle X satisfies  $E_X^2 = \mathbf{p}_X^2 + m_X^2$ , and is called “on-mass shell”. Similarly for the right hand side diagram on slides 3 and 4, we have

$$M_{fi}^{ba} = \frac{1}{2E_X} \cdot \frac{g_a g_b}{(E_b - E_d - E_X)}.$$



To obtain total amplitude of particle interaction, different amplitudes for a process are summed, giving

$$M_{fi} = M_{fi}^{ab} + M_{fi}^{ba} = \frac{g_a g_b}{2E_X} \cdot \left( \frac{1}{E_a - E_c - E_X} + \frac{1}{E_b - E_d - E_X} \right).$$

Applying conservation of energy  $E_b - E_d = E_c - E_a$ , we have

$$M_{fi} = \frac{g_a g_b}{(E_a - E_c)^2 - E_X^2}.$$

For both diagrams the exchanged particles momenta is related by the Einstein energy-momenta relation  $E_X^2 = \mathbf{p}_X^2 + m_X^2$ . Momentum is conserved at each interaction vertex giving  $\mathbf{p}_X = (\mathbf{p}_a - \mathbf{p}_c)$  for the first (left) time ordered process and  $\mathbf{p}_{\bar{X}} = (\mathbf{p}_b - \mathbf{p}_d) = -(\mathbf{p}_a - \mathbf{p}_c)$  for the second (right) time ordered process. For both processes energy of the exchanged particle can then be written  $E_X^2 = \mathbf{p}_X^2 + m_X^2 = (\mathbf{p}_a - \mathbf{p}_c)^2 + m_X^2$ . The matrix element becomes

$$M_{fi} = \frac{g_a g_b}{(p_a - p_c)^2 - m_X^2}$$

Where  $p_a$  and  $p_c$  are the respective four-momenta of particles  $a$  and  $c$ . Writing the four-momentum of the exchanged virtual particle  $X$  as  $q = p_a - p_c$ , we have

$$M_{fi} = \frac{g_a g_b}{q^2 - m_X^2}.$$

The term  $\frac{1}{q^2 - m_X^2}$  is referred to as the propagator and is associated with the exchanged particle with terms  $g_a$  and  $g_b$  associated with the interaction vertices. ■

Cross section for the production of two jets from t-channel gluon to gluon exchange process  $qq \rightarrow qq$  is :

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_s^2}{9Q^4} \left[ 1 - \left( 1 - \frac{Q^2}{\hat{s}} \right)^2 \right]$$

where  $Q^2 = -q^2$  and  $\hat{s} = x_1 x_2 s$  is the center of mass energy of the colliding quarks. In terms of PDFs :

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_s^2}{9Q^4} \left[ 1 + \left( 1 - \frac{Q^2}{sx_1x_2} \right)^2 \right] g(x_1, x_2) dx_1 x_2$$

where  $g(x_1, x_2)$  is the sum over the products of the relevant PDF process for scattering  $qq \rightarrow qq$ .

Conservation of energy and momenta implies

$$x_1 = \frac{p_T}{\sqrt{s}} (e^{+y_3} + e^{+y_4}) \quad \text{and} \quad x_2 = \frac{p_T}{\sqrt{s}} (e^{-y_3} + e^{-y_4})$$

$$\text{Hence, } Q^2 = p_T^2 (1 + e^{y_4 - y_3})$$

